UDC 533.6.011

NEIGHBORHOOD OF THE LEADING EDGE OF A PLANE POINTED PROFILE IN SUPERSONIC FLOW

PMM Vol. 43, No. 3, 1979, pp. 480-488
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(Received April 19, 1978)

The local structure of flow near the singular point of a pointed profile in a supersonic stream of non-heat-conducting gas with a shock wave attached to the profile leading edge. Gasdynamic functions are taken in the form $f = f_0 + f_1 + o(f_1)$, where f_0 corresponds to the flow past an infinite rectilinear wedge. Linearization with respect to f_0 results in some boundary value problem in an angle. Its solution is obtained in explicit form, and its properties are investigated. It is shown that when f_0 relates to supersonic flow, then $f_1 \equiv 0$. In the case of subsonic flows a sequence of nonzero eigenfunctions $f_1^{(m)}$ are obtained. The first $f_1^{(m)}$ are considered in transonic formulation. Difference between "strong" and "weak" shock waves is established. Both, homogeneous (rectilinear profile and uniform oncoming stream) and inhomogeneous cases are investigated.

1. Let a smooth pointed profile be placed in a supersonic stream. We assume the upper and lower shock waves attached to the profile, hence they can be considered independently. We shall use the notation: P for pressure, ρ for density, M for the Mach number, q for the velocity modulus, and θ for the angle of inclination of velocity to the x-axis in the direction of flow. Let α be the angle of inclination of the shock wave at the leading edge and β the angle of the profile at that point. Pressure and density are normalized with respect to the corresponding parameters of the oncoming stream whose Mach number we denote by M_∞.

We shall use equations of gasdynamics of the form

$$\rho q^2 \frac{\partial \theta}{\partial s} + \frac{\partial P}{\partial n} = 0 \tag{1.1}$$
$$\rho q^2 \frac{\partial \theta}{\partial n} + (1 - M^2) \frac{\partial P}{\partial s} = 0$$

where $\partial/\partial s$ and $\partial/\partial n$ are derivatives along streamlines and perpendicular to them.

We represent the solution in the neighborhood of the leading edge in the form $f = f_0 + f_1 + \ldots$, where f_0 is the main part which defines the flow past an infinite wedge with a plane shock wave attached to it. Generally $f_0 = f_0(\varphi)$ (φ is the polar angle). But the functions in system (1.1) do not depend on φ, hence for small perturbations, after the substitution $\xi = s_0$, $\eta = n_0 |1 - M_0^2|^{1/2}$, and $\sigma_1 = P_1 |1 - M_0^2|^{1/2} / (\rho_0 q_0^2)$, where s_0 and n_0 are rectangular coordinates (see Fig. 1), we obtain

$$\frac{\partial \theta_1}{\partial \xi} + \frac{\partial \sigma_1}{\partial \eta} = 0, \quad \frac{\partial \theta_1}{\partial \eta} - \operatorname{sign}(1 - M_0^2)\frac{\partial \sigma_1}{\partial \xi} = 0 \qquad (1.2)$$

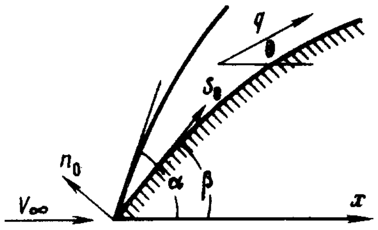

Fig. 1

The region of solution determination is bounded by the half-lines $\eta = 0$ (the body) and $\eta = k\xi$ (wave), where $k = |1 - M_0^2|^{1/2} \operatorname{tg}(\alpha - \beta)$. Let us determine the boundary conditions at the wave.

Let the polar equation of the shock wave be of the form $\varphi = \alpha + \delta(r)$, where φ is the wave angle of inclination to the x-axis. Then $\varphi = \alpha + \delta + \operatorname{arctg}(r\delta_r)$. In the neighborhood of the leading edge $\varphi = \alpha + (r\delta)_r$. The value of any gasdynamic function $f(r, \varphi)$ along the wave is determined by its inclination $f = f_0(\varphi) + f_1(r, \varphi) = F(\psi)$. Neglecting the term $\delta \partial f_1/\partial\varphi$, we obtain

$$f_0(\alpha) + \delta\frac{\partial f_0}{\partial \varphi} + f_1(r, \alpha) = F(\alpha) + \left(\frac{dF(\alpha)}{d\psi}\right)(r\delta)_r$$

Since $f_0(\alpha) = F(\alpha)$, hence $f_1(r, \alpha) = -\delta\partial f_0/\partial\varphi + (dF(\alpha)/d\varphi)(r\delta)_r$. Thus $P_1(r, \alpha) = (dP(\alpha)/d\psi)(r\delta)_r$ and $\theta_1(r, \alpha) = (d\theta(\alpha)/d\psi)(r\delta)_r$, from which

$$\frac{dP}{d\psi}\theta_1(r, \alpha) = \frac{d\theta}{d\psi}P_1(r, \alpha), \quad \psi = \alpha \qquad (1.3)$$

Using the relations at the shock wave

$$P = [2\gamma M_\infty^2 \sin^2\psi - (\gamma - 1)] / (\gamma + 1)$$

$$\operatorname{ctg}\theta = \operatorname{tg}\psi\left[\frac{(\gamma + 1) M_\infty^2}{2(M_\infty^2 \sin^2\psi - 1)} - 1\right]$$

from (1.3) we derive for system (1.2) the required boundary condition

$$A\sigma_1 - B\theta_1 = 0 \qquad (1.4)$$

$$A = \rho_0 q_0^2 \left[2\zeta_0(1 + \zeta_0)^2 \operatorname{ctg}^2\theta_0 - \frac{\gamma + 1}{2}(1 + \zeta_0^2 \operatorname{ctg}^2\theta_0)\right] \times (\gamma M_\infty^2 |1 - M_0^2|^{1/2})^{-1}$$

$$B = 2\zeta_0^2 \operatorname{ctg}\theta_0 \sin^{-2}\theta_0, \quad \zeta^{-1} = \frac{\gamma M_\infty^2}{P - 1} - 1$$

The boundary condition at the body will be derived later.

The considered boundary value problem has two dimensionless parameters: M_∞ and β. We fix M_∞ and analyze the dependence on the "geometrical" parameter for which we take not β but α, since the dependence of the unperturbed solution (flow past a wedge) on β is not uniquely defined. The variation range of α is from $\arcsin(M_\infty^{-1})$ (degenerate shock wave) to $\pi/2$ (plane normal wave).

We denote by α_* the value of α for which $M_0 = 1$ and by α^* the angle which corresponds to the maximum turn of the (unperturbed) stream, i.e. at the point of transition from the weak section of the shock polar to the strong one. Clearly $\alpha_* < \alpha^*$. We introduce function $h(\alpha) = A / B$ some of whose properties will be subsequently required. They are obtained by simple calculations. We have $h(\alpha) > 0$ when $\alpha < \alpha^*$, $h(\alpha) < 0$ when $\alpha > \alpha^*$, and $h(\alpha^*) = 0$, and $h(\alpha_*) = +\infty$.

Note that the boundary condition (1.4) may also be written in the form

$$\partial\theta_1/\partial l = 0 \tag{1.5}$$

where l is at angle χ to the boundary such that $\operatorname{tg}\chi = h$.

2. Let us define the boundary condition at the profile surface in some neighborhood of the leading edge where the profile is assumed to be rigorously rectilinear. We have there $\theta = \beta$ and

$$\theta_1(r, \beta) = 0 \tag{2.1}$$

We thus obtain the homogeneous problem (1.2), (1.4), (2.1).

Let us first consider the case of supersonic flow ($M_0 > 1$, i.e. $\alpha < \alpha_*$. Obviously

$$\theta_1 = f(\xi + \eta) - f(\xi - \eta), \quad \sigma_1 = C - f(\xi + \eta) - f(\xi - \eta)$$

$$\varkappa f(z) + f(\mu z) = \frac{\varkappa + 1}{2} C, \quad z = (1 + k)\xi, \quad \varkappa = \frac{h + 1}{h - 1}$$

$$\mu = \frac{1 - k}{1 + k}$$

where C is an arbitrary constant. Substituting function $F(z) = f(z) - C/2$, for $f(z)$, we obtain the functional equation

$$F(\mu z) = -\varkappa F(z) \tag{2.2}$$

Let us estimate μ and $\varkappa$. Since $h > 0$, hence $|\varkappa| > 1$ and the inclination of the related characteristic is greater than that of the shock wave, i.e. $(M_0^2 - 1)^{-1/2} > \operatorname{tg}(\alpha - \beta)$, hence $0 < k < 1$ and, consequently $0 < \mu < 1$.

Iterations (2.2) yield $F(\mu^n z) = (-\varkappa)^n F(z)$. We pass to limit for $n \to \infty$. Since $\mu^n \to 0$ and $\varkappa^n \to \infty$ hence in the class of functions bounded in the neighborhood of the leading point Eq. (2.2) has the unique (trivial) solution $F(z) \equiv 0$. Consequently $\theta_1 = \sigma_1 \equiv 0$ and the shock wave is rectilinear.

3. We pass to the case of subsonic flow ($M_0 < 1$) in which (1.2) is converted to the Cauchy—Riemann type system. For its solution it is convenient to pass to coordinates (R, Φ): $\xi = R\cos\Phi$ and $\eta = R\sin\Phi$. Obviously $\theta_1 = R^\nu \sin \nu\Phi$, where $\operatorname{tg}(\nu \operatorname{arctg} k) = h$, i.e.

$$\nu = \nu_m = \frac{\pi m + \operatorname{arctg} h}{\operatorname{arctg} k} \tag{3.1}$$

Because of the boundedness of solution only positive ν_m ought to be selected. The whole sequence of ν_m "originates at infinity" when $\alpha = \alpha_*$. As α increases, only ν_0 passes through zero. The remaining ν_m are positive, as implied by the properties of h. Curves of several first ν_m at $M_\infty = 3$ are shown in Fig. 2, where the negative part of ν_0 is represented by the dash line. Note that $\nu_m(\pi/2) = 2m-1$. Thus

$$\theta = \theta_0 + CR^{\nu}\sin \nu\Phi + \ldots \qquad (3.2)$$

$$P = P_0 + C\frac{\rho_0 q_0^2}{\sqrt{1-M_0^2}} R^{\nu}\cos\nu\Phi + \ldots$$

For determining density we use the equation $\partial S/\partial s = 0$; where $S = P\rho^{-\gamma}$ and $S = S_0 + S_1 + \ldots$. It can be shown that $S_1 = \rho_0^{-\gamma}(P_1 - c_0^2\rho_1)$.

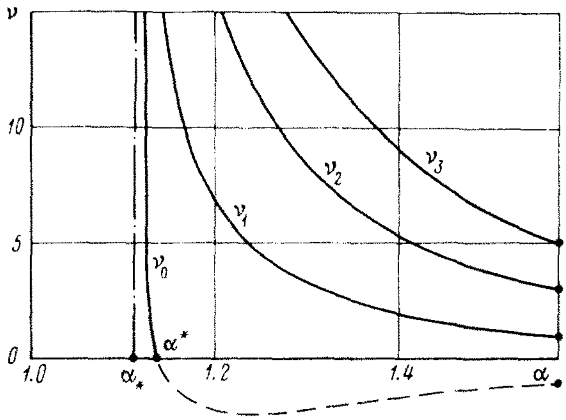

Fig. 2

At the shock wave $\rho_1^{(w)} = D_1 P_1^{(w)}$ and $P_1^{(w)} = D_2 R^{\nu}$, hence $S_1^{(w)} = D_3 R^{\nu}$. Since $S_1 = f(\eta) = f(R\sin\Phi)$, hence finally $S_1 = D_3(R\sin\Phi)^{\nu}$, from which

$$\rho = \rho_0 + R^{\nu}(a_1\cos\nu\Phi + a_2\sin^{\nu}\Phi) + \ldots \qquad (3.3)$$

Velocity q is obtained with the use of variations of the Bernoulli integral

$$q = q_0 + R^{\nu}(a_3\cos\nu\Phi + a_4\sin^{\nu}\Phi) + \ldots \qquad (3.4)$$

When θ_1 is known, it is possible to show that in the neighborhood of the leading edge the wave equation can be represented in the form

$$y = x \operatorname{tg}\alpha + x^{1+\nu} + \ldots$$

We stress the difference in the behavior of gasdynamic functions when $\Phi \to 0$. It is particularly noticeable when $\nu < 1$, since then ρ_1 and q_1 are no longer differentiable along the profile surface.

Formulas (3.2)—(3.4) provide a solution of the problem with an accuracy within the specific exponent ν_m which is dictated by the problem as a whole. The number of zeros of $P_1(\varphi)$ (which coincides with the number of internal extremal points $\theta_1(\varphi)$) is equal to the number m, while the number of zeros of $\theta_1(\varphi)$ is by one smaller of the ordinal number of the exponent in the sequence of positive ν, i.e. in the interval $\alpha_* < \alpha < \alpha^*$ the number of zeros of θ_1 is equal m and when

$\alpha > \alpha^*$ it is equal $m - 1$, since ν_1 becomes the first positive exponent. Thus the number of zeros uniquely determines ν_m and, consequently, also the first term of expansion in the neighborhood of the singularity. In certain cases such information may be obtained.

The quantity ν defines the smoothness of $\theta_1 (R, \Phi)$. In connection with this it is possible to isolate the value $\alpha = \alpha_K$ $(\alpha_* < \alpha_K < \alpha^*)$ for which $\nu_0 = 1$. The corresponding point of the shock wave is called the Crocco point. It should be noted that the shock wave curvature at the leading edge if finite when $\alpha = \alpha_K$, vanishes when $\alpha < \alpha_K$, and becomes infinite when $\alpha > \alpha_K$. Specific values of the basic "support" angles α_*, α_K and α^* and the related values of β are tabulated below

M_∞	α_*	$\beta_* \cdot 10^4$	α_K	$\beta_K \cdot 10^4$	α^*	$\beta^* \cdot 10^4$
1.25	1.1590	870	1.1936	908	1.2311	923
1.5	1.0866	2041	1.1274	2098	1.1622	2114
2.0	1.0731	3963	1.1076	4003	1.1287	4010
2.5	1.0934	5178	1.1182	5198	1.1307	5201
3.0	1.1129	5936	1.1308	5946	1.1387	5947
4.0	1.1389	6764	1.1491	6767	1.1530	6767
5.0	1.1534	7175	1.1599	7176	1.1621	7176
7.5	1.1693	7600	1.1721	7601	1.1730	7601
10.0	1.1752	7754	1.1768	7754	1.1773	7754
20.0	1.1812	7905	1.1816	7905	1.1817	7905

Let us now touch upon the fallacy shared by some authors (see, e.g., [3]). Vanishing of the exponent ν_0 at transition from the weak section of the shock wave to the strong one can neither be a proof nor even an argument to support the theory of non-existence of an attached shock wave of the strong kind. Generally speaking, the expansion may begin with any of the possible exponents, and the whole difference between strong and weak shock waves lies in the structure of small perturbations of θ_1 and P_1.

4. We shall illustrate the above results on some transonic flows, using for this the hodograph plane u, v (u is the velocity component along the x-axis and v the component perpendicular to it). We shall consider a problem of the type of Frankl's problem [4,5]— the flow past a wedge of finite dimensions—or, more exactly, that of its varieties in which the shock wave is attached to the leading edge.

In such problems the investigation of the leading edge neighborhood in the physical plane is equivalent to the investigation of the neighborhood of point O_1 of intersection of the shock polar w with the straight line b ($v = u \operatorname{tg} \beta$). There are generally two such points (see Fig. 3).

Motion in the physical plane along some small arc $d_\varepsilon : \{r = \varepsilon, \beta < \varphi < \alpha\}$ from the body contour to the wave is represented in the hodograph plane by the motion of the corresponding point in the neighborhood of O_1 (or O_2) from the straight line b to the polar w. Its polar angle $\theta = \operatorname{arctg} (v / u)$ differs from $\theta_0 = \beta$ by some small quantity which with an accuracy within the constant multiplier coincides

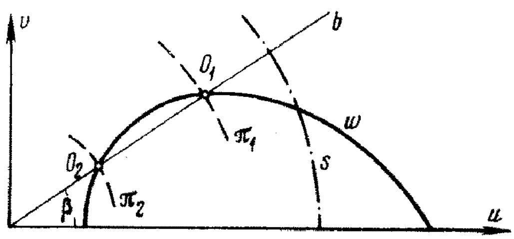

Fig. 3

with the solution of the problem of small perturbations, i.e. $\theta = \theta_0 + C\theta_1$.

We plot in the hodograph plane the lines that are images of isobars (π_1 and π_2 in Fig. 3, where s is the sonic line). In transonic approximation these lines coincides with lines of constant values of the velocity modulus, and are, thus, circles whose centre is at the coordinate origin. In what follows only their orthogonality to line w is important. Since the isobars pass through O_i, the perturbations of P_1 on them vanish.

Let the neighborhood $\omega : \{r < \varepsilon,\ \beta < \varphi < \alpha\}$ be mapped onto some region Ω. We shall show that the angular dimension of Ω depends on the ordinal number of the exponent ν, using for this the dependence of the number of zeros of $P_1(\varphi)$ on m. Let us, first, consider the weak shock wave (O_1). Let $\nu = \nu_0$. Then $P_1 \neq 0$, the image of d_ε does not intersect π_1 and, consequently, the aperture angle of Ω is acute. When $\nu = \nu_1$ that angle is in the interval of π to ${}^3/_2\pi$. If $m > 1$, the motion along d_ε is represented in the hodograph plane by the motion of a point which goes around O_1 more than once, because of which the inverse transformation becomes ambiguous.

It is possible to show in exactly the same way that in the case of a strong wave (O_2) an obtuse aperture angle of region Ω corresponds to the first positive exponent (which is now ν_1), and to the second an angle comprised in the interval of ${}^3/_2\pi$ to 2π. Ambiguity appears with higher ν_m.

The first two positive exponents, thus, have some kind of priority, since to them correspond one-to-one transformations of the hodograph (in the leading edge neighborhood). There remains, however, the ambiguity in the selection of one of the two exponents which in this case can only be removed by considering the problem as a whole. The flow past a double wedge when the Mach number in the space between the wedge and the shock wave is less than unity (Fig. 4, a).

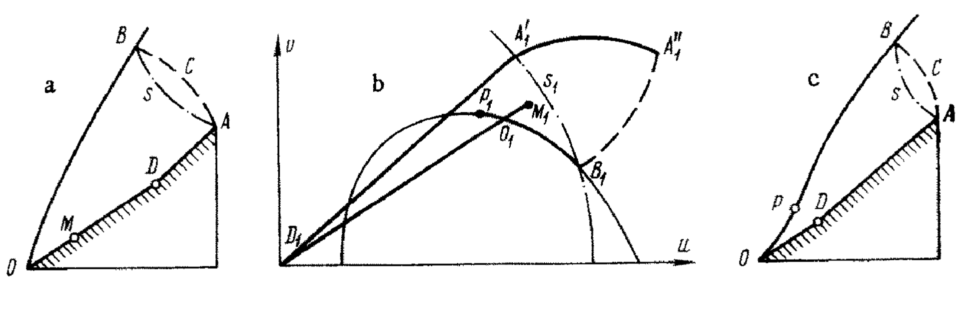

Fig. 4

If the length OD is fairly large (as compared with DA), the velocity along it is not monotonic, first it increases, reaches maximum at some point M, and then decreases vanishing at D. In this case M_1, the image of point M, is on the continuation of segment D_1O_1 between the shock polar and the sonic line s_1 (Fig. 4, b). Region $ACBOD$ passes to region with slit $A_1'A_1''B_1O_1M_1D_1$. As OD decreases, the slit O_1M_1 decreases and at some instant altogether vanishes (the second case). Further decrease of OD yields regions of the type of $A_1'A_1''B_1P_1O_1D_1$ where there is another slit O_1P_1 along the shock polar (the third case). Its length increases in proportion to the decrease of OD. The double passage on the polar obviously shows the presence of an inflection point on the shock wave (Fig. 4, c).

In this example the neighborhood of point O_1 is interesting. In the first case the angle $B_1O_1M_1$ is acute, in the second it is obtuse, and in the third it is again acute (but situated on the other side of point O_1). This means that the expansion of function θ in the neighborhood of the leading edge is of the form $\theta = \theta_0 + CR^{\nu_0} \sin \nu_0\Phi + \ldots$, where θ_0, ν_0 and C depend on the position of point D. When D is in a neutral position, $C(D)$ passes through O. At that instant the first term of expansion (different from θ_0) vanishes and becomes included in the next following term which is related to the region of the obtuse angle, as shown above, in the hodograph plane.

A similar situation occurs in the flow past a double wedge in a channel when the larger angle exceeds the critical value. The same considerations apply to the problem of flow past a straight wedge in a channel. It is clear that the selection of structure of solution in the neighborhood of the leading edge can only be made after assessment of the problem as a whole.

5. Let us revert to the initial statement for system (1.2) but consider now the inhomogeneous problem. We assume the equation of the contour in the leading edge neighborhood to be of the form $y = x \operatorname{tg} \beta + ax^{1+s} + \ldots, s > 0$. For the slope of the streamline along the contour we have $\theta = \beta + a(1+s)\, r^s \cos^{2+s} \beta$. Hence the boundary condition (2.1) is replaced by $\theta_1 = Cr^s$ when $\varphi = \beta$.

In the supersonic case $(M_0 > 1)$ the passage to coordinates (ξ, η) yields $\theta_1 = C\xi^s$, and the solution is defined by formulas

$$\theta_1 = C[\lambda(\xi+\eta)^s + (1-\lambda)(\xi-\eta)^s] \tag{5.1}$$
$$\sigma_1 = C[-\lambda(\xi+\eta)^s + (1-\lambda)(\xi-\eta)^s], \quad \lambda = \mu^s / (\varkappa + \mu^s)$$

The constants μ and $\varkappa$ have been determined in Sect. 2 and, consequently, satisfy the inequalities $|\varkappa| > 1$, $0 < \mu < 1$, which ensures the finiteness of λ, i.e. the existence of solution throughout the range of $\alpha < \alpha_*$. The uniqueness (triviality of solution of the homogeneous problem) has been already proved.

For $\alpha > \alpha_*$ it is necessary to pass to coordinates R, Φ in which the boundary conditions on the contour (for $\Phi = 0$) are $\theta_1 = CR^s$, and for θ_1 and σ_1 we obtain

$$\theta_1 = CR^s(\cos s\Phi + D \sin s\Phi)$$
$$\sigma_1 = CR^s(-\sin s\Phi + D\cos s\Phi) \tag{5.2}$$

$$D = (h \operatorname{tg} s\Phi^{(w)} + 1) / (h - \operatorname{tg} s\Phi^{(w)})$$

Thus formula (5.2) provides the solution of the problem in the general case, i.e. when $h \neq \operatorname{tg} s\Phi^{(w)}$. However it is not unique, since there exists, as previously shown, a sequence of nontrivial functions θ_1 and σ_1. Hence the obtained result needs clarification.

Let s be smaller than the first positive exponent ν. Then formulas (5.2) actually provide the first expansion term. If, however, $s > \nu$, the derived solution must be discarded, since the homogeneous problem brings stronger perturbations. From the geometrical point of view this means that in the first approximation with $s < \nu$ the flow reacts to the profile curvature, while with $s > \nu$ does not. The first to note this effect was Guderley [6] who came across it in transonic flow investigations.

Let us consider the particular case of $h = \operatorname{tg} s\Phi^{(w)}$, in which a solution of the form (5.2) is not possible, and the expansion must contain logarithmic terms. For its derivation we use the harmonicity of $\theta_1(R, \Phi)$. We pass from the plane $z = Re^{i\Phi}$ to the plane $w = z^s = R_1 \exp(i\Phi_1)$, with $R_1 = R^s$ and $\Phi_1 = s\Phi$.

Function $g(R_1, \Phi_1) = \theta_1(R, \Phi)$ is also harmonic. We represent the boundary condition at the shock wave in the form of equality to zero of the oblique derivative (1.5). Owing to the conformality of transformation the angle χ between the direction of differentiation and the boundary does not change, only the boundary itself turns $\Phi = \Phi^{(w)} \to \Phi_1 = \Phi_1^{(w)} = s\Phi^{(w)}$. The condition of impermeability is simplified

$$g(R_1, 0) = R_1 \tag{5.3}$$

Let us determine g. We have $\operatorname{tg}\chi = h$ and $\operatorname{tg}\nu\Phi^{(w)} = h = \operatorname{tg} s\Phi^{(w)} = \operatorname{tg}\Phi_1^{(w)}$. Hence $\Phi_1^{(w)} = \chi$ when $\alpha < \alpha^*$ or $\Phi_1^{(w)} = \pi + \chi$ when $\alpha > \alpha^*$. In both cases the direction of differentiation is now parallel to the boundary $\Phi_1 = 0$. We introduce the coordinates $\xi_1 = R_1 \cos\Phi_1$, $\eta_1 = R_1 \sin\Phi_1$ and reduce the problem to the determination of function $g(R_1, \Phi_1)$ harmonic in the angle $0 < \Phi_1 < \Phi_1^{(w)}$ which satisfies the boundary conditions

$$g(R_1, 0) = \xi_1, \quad \left.\frac{\partial g}{\partial \xi_1}\right|_{\Phi_1 = \Phi_1^{(w)}} = 0$$

We pass from g to $\Theta = \partial g/\partial\xi_1$. The boundary conditions for Θ are $\Theta = 1$ when $\Phi_1 = 0$ and $\Theta = 0$ when $\Phi_1 = \Phi_1^{(w)}$. Hence

$$\Theta = 1 - \frac{\Phi_1}{\Phi_1^{(w)}} = 1 - \frac{1}{\Phi_1^{(w)}} \operatorname{arctg}\frac{\eta_1}{\xi_1} = 1 - \frac{\pi}{2\Phi_1^{(w)}} + \frac{1}{\Phi_1^{(w)}} \operatorname{arctg}\frac{\xi_1}{\eta_1}$$

Reverting to g we obtain

$$g = C(\eta_1) + \xi_1\left(1 - \frac{\pi}{2\Phi_1^{(w)}}\right) + \frac{1}{\Phi_1^{(w)}}\left[\xi_1 \operatorname{arctg}\frac{\xi_1}{\eta_1} - \frac{\eta_1}{2}\ln\left(1 + \frac{\xi_1^2}{\eta_1^2}\right)\right]$$

$$C(\eta_1) = A + B\xi_1 - \frac{\eta_1}{\Phi_1^{(w)}}(\ln \eta_1 - 1)$$

where the term $C(\eta_1)$ is determined by the harmonicity of g.

The boundary condition at the body yields $A = B = 0$. Reverting to R_1 and Φ_1 and then to R and Φ, we finally obtain

$$\theta_1 = aR^s \ln R + bR^s \tag{5.4}$$

$$a = -\frac{1}{\Phi^{(w)}} \sin s\Phi, \qquad b = \left(1 - \frac{\Phi}{\Phi^{(w)}} \cos s\Phi + \frac{1}{s\Phi^{(w)}} \sin s\Phi\right.$$

The formula for $\sigma_1(R, \Phi)$ is similarly derived. The logarithmic term is also contained in the wave equation.

Note that in the obtained asymptotics both terms must be retained in spite of the fact that the first tends to zero more slowly. The point is that their derivatives with respect to R are of the same order.

In concluding we shall consider the effect of inhomogeneity of the oncoming stream. For simplicity we confine this to the case of local linearity in the leading edge neighborhood and a rectilinear profile. The system of equations for small perturbations and the condition on the body do not differ from those of the homogeneous problem, while conditions at the shock wave assume the form

$$A\sigma_1 - B\theta_1 = Cr = C_1\xi = C_2\eta, \quad \eta = k\xi$$

It is obvious that when $\nu \neq 1$ the principal term of expansion is obtained from the solution of the inhomogeneous problem

$$\theta = R(D_1 \sin \Phi + D_2 \cos \Phi), \quad \nu > 1$$

or from that of the homogeneous problem

$$\theta = R^\nu D_3 \sin \nu\Phi, \quad \nu < 1$$

If, however, $\nu = 1$, then by passing from θ_1 to Θ we obtain a problem whose solution $\Theta = C\Phi_1$ is elementary. From this

$$\theta_1 = D_1(\Phi) R \ln R + D_2(\Phi) R$$

which for $s = 1$ coincides with (5.4) with an accuracy to the coefficients.

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